Steiner Problem & Networks

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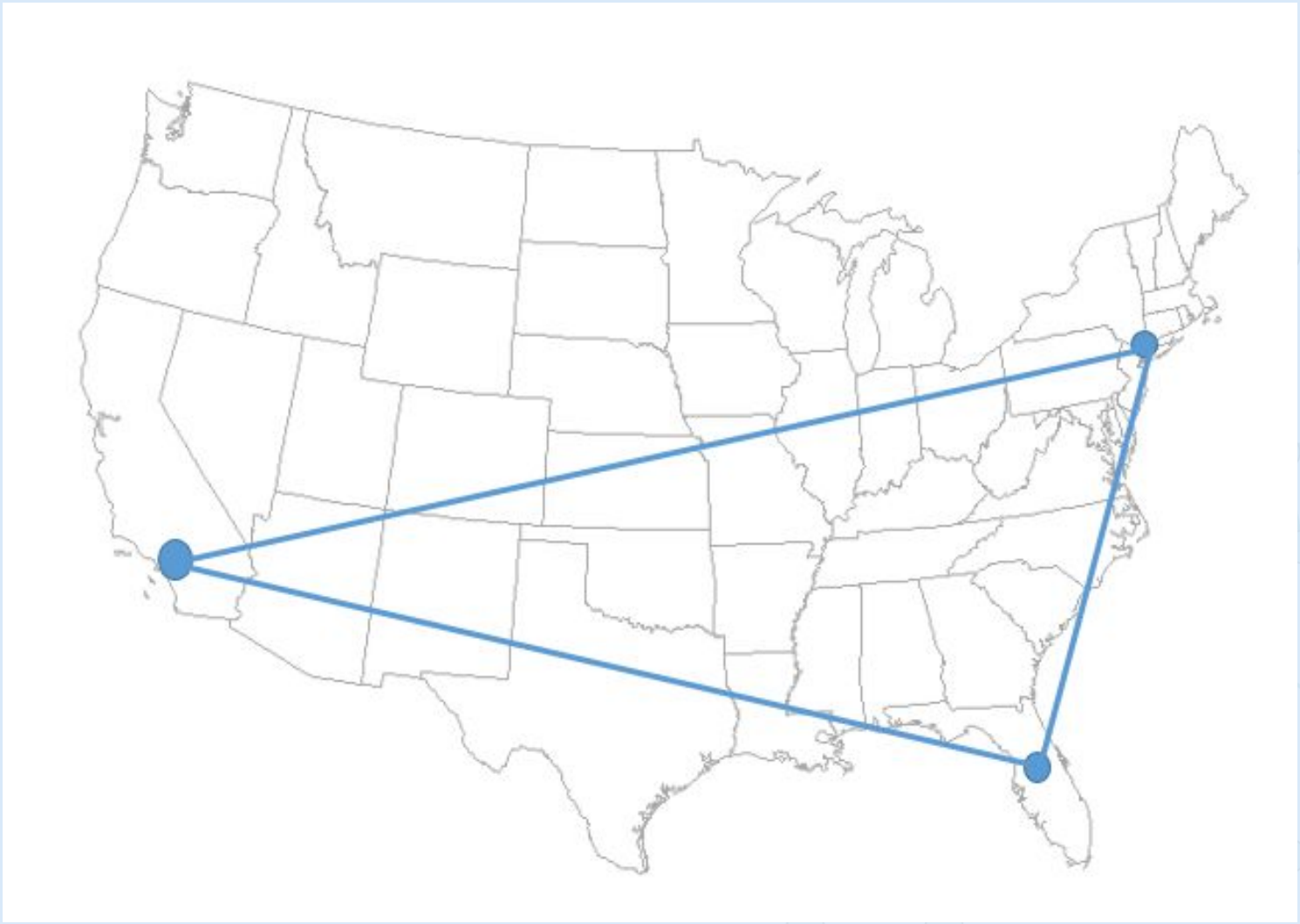
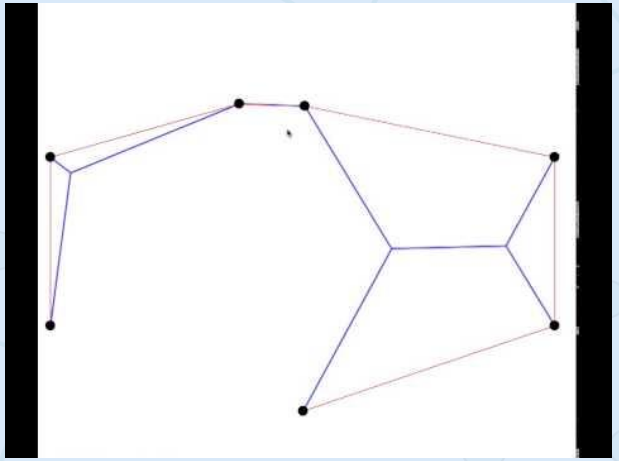


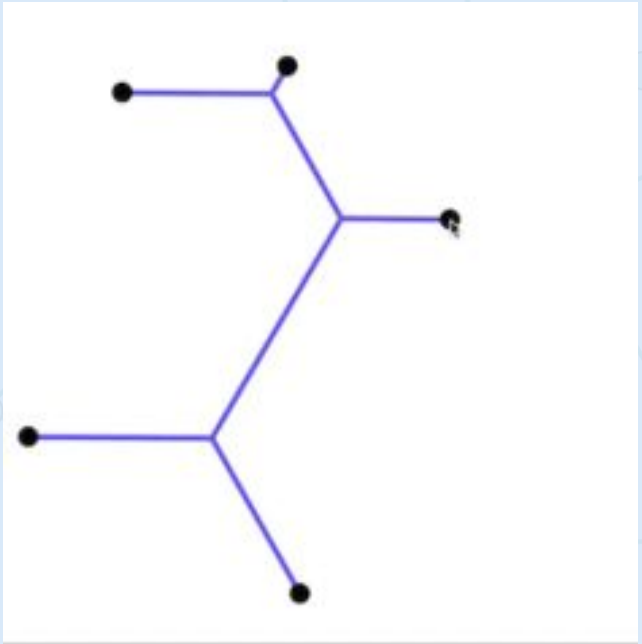
What is the Steiner Tree Problem?

- If N is a set of points in the plane, the Euclidean Steiner Tree Problem is: Find a set of line segments so that all the points are connected with each other and so that the total Euclidean length of the line segments is minimized.
    - Extra points a.k.a. Steiner Points
    - Solution: a minimum steiner tree
- Steiner Trees are considered more efficient than minimum spanning trees.



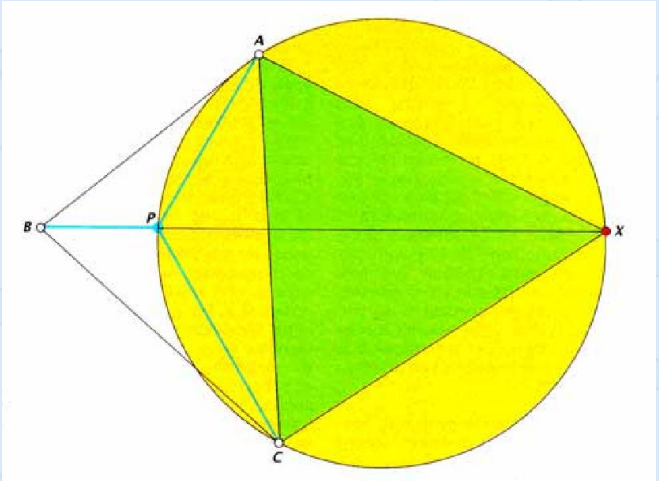
Properties of Steiner Points

- Always have degree 3
- Edges that meet at 120° at the point
- No more than n - 2 steiner points in any n-point graph
- Trees that use all n-2 points are called full steiner trees
- When n is greater than 3, the problem will be reduced to cases within the original graph.



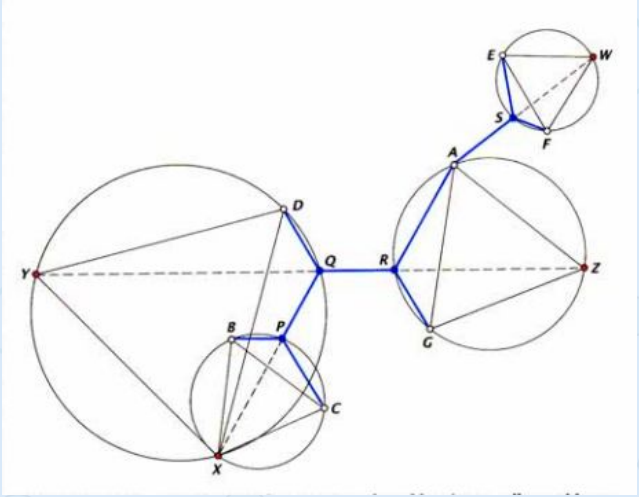
Algorithm for three point problem

- Geometric way- Fermat-Torricelli point
  - Determine largest angle. Is it less than 120? Then we are going to use this algorithm
  - Take the largest length and create an equilateral triangle
  - Circumscribe the triangle and connect the third vertex to the third point of the original graph.
  - The intersection of the circle and the line creates a Steiner point
- Computers simplify our life and can do this for us.



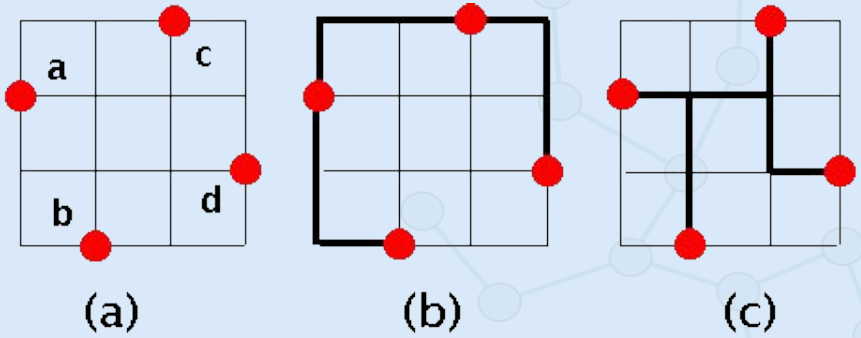
More than three points

- Break up into smaller parts using Melzak's algorithm
- Determining which points to use first gets a little tricky
  - Computers can come into play but even they take time
- The Steiner Problem is a NP-hard problem
  - It's considered a complex problem
  - Optimal solution may not be found in polynomial time algorithm ("fast" algorithm)



Application in Networks

- Rectilinear Steiner Trees
  - Graph is on a computer chip grid to increase operating speeds
  - On a grid were not all properties of the Euclidean Steiner Tree problem apply
- Networks
  - Telephone companies
  - Pipelines
  - Roadways
  - Computer chips

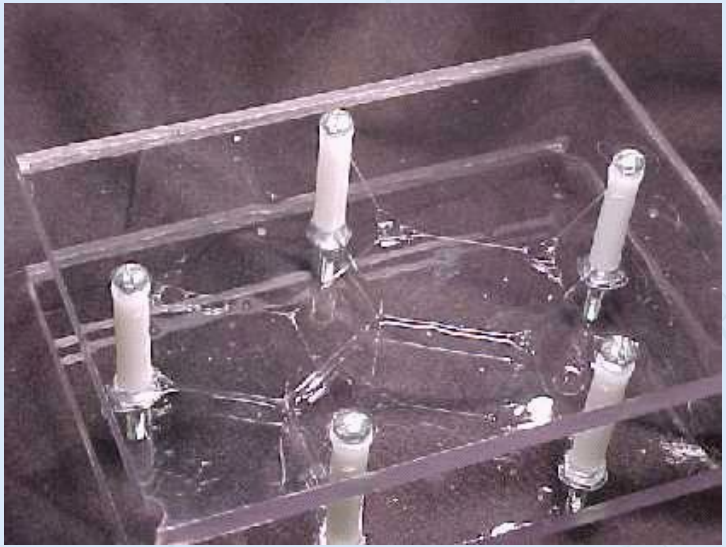


http://cs.engr.uky.edu/~lewis/research/Papers/Steiner-Local/Image14.gif

http://www.ecritel.fr/IMG/jpg/network2.jpg

Endnotes

- Can be solved through:
  - Meticulous human calculated solutions
  - Computers
  - Soap bubbles
- Steiner Problem in the 3rd dimension
  - Applications in space



http://www.pandasthumb.org/archives/2006/07/target_target_w_1.html

Resources

http://www.math.ucsd.edu/~ronspubs/89_01_shortest_network.pdf

http://www.science.ku.dk/english/press/news/2014/the-steiner-tree-research-at-diku-experiences-a-revival/

http://onlinelibrary.wiley.com/doi/10.1002/net.3230170203/pdf

http://demonstrations.wolfram.com/SteinerNetworksForFourPoints/

http://demonstrations.wolfram.com/ConstructingASteinerTreeForFivePoints/

https://www.encyclopediaofmath.org/index.php/Steiner_tree_problem

https://archive.org/details/RonaldLG1988