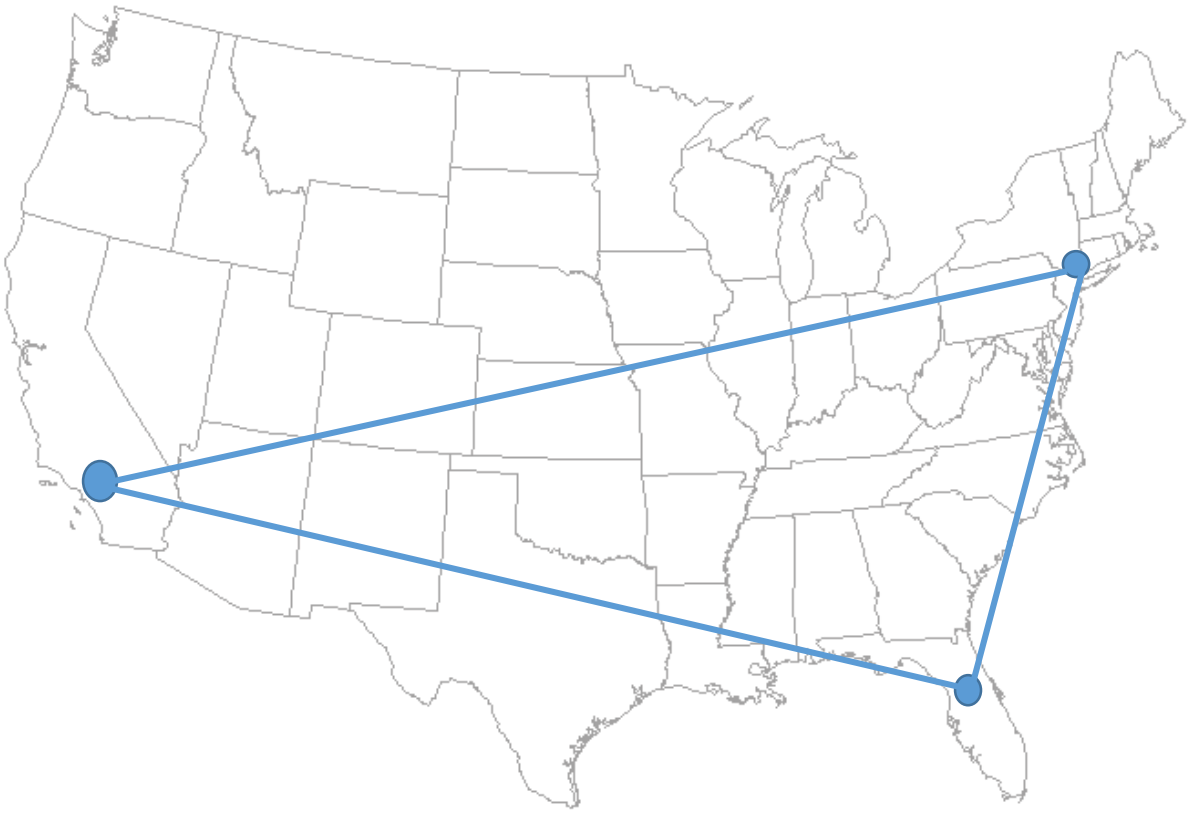
Name _____ Date _____

Wkst Created by Toni Navarro

You are a part of the Knights of Win Racing Company. Your superiors have told you to look into the connections between three of the main central offices across the United States. There is one in Los Angeles, California, one in New York City, New York, and one in Orlando Florida. The company would like to make most cost efficient network between the places. If the network costs $1 per mile between the cities, what connections should they keep to ensure that all offices are connected but for the cheapest total cost?

Distance between NYC and Los Angeles: 2451 miles
Distance between NYC and Orlando: 940 miles
Distance between Orland and Los Angeles: 2200 miles



Is this truly the cheapest? What if the company decided to open up a new location?

Where could you add a new location in the middle of the triangle created by the other central offices that would cause the network to be even cheaper than it is now?

The Steiner Tree Problem:

If N is a set of points in the plane, the Euclidian Steiner Tree Problem is: Find a set of line segments so that all the points are connected with each other and so that the total Euclidian length of the line segments is minimized.

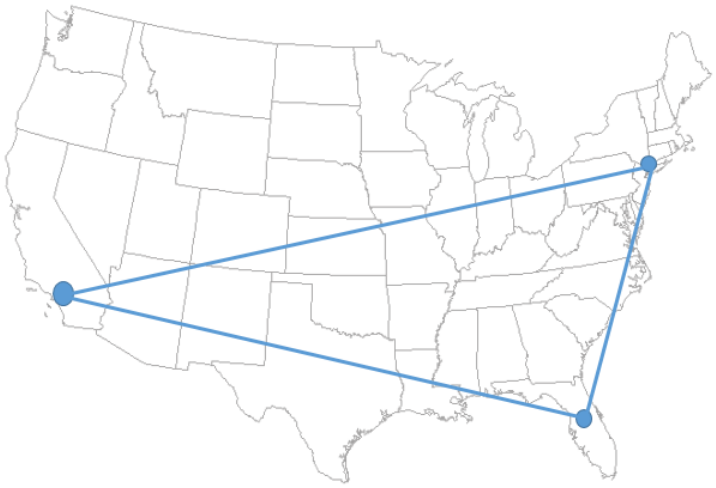
Steiner Points – points that are added to a graph to create a minimum spanning tree

Steiner Tree – the solution to the Steiner Tree Problem where the endpoints of the tree are points from the original graph.

Properties of Steiner points:

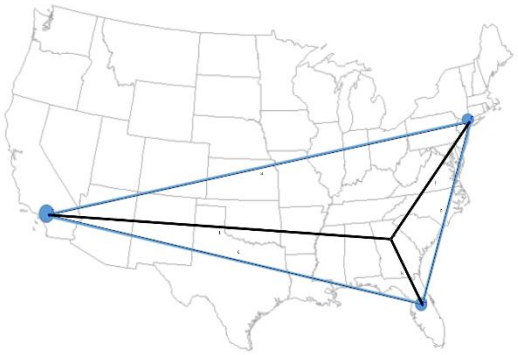
- Always have degree of three
- The edges that connect to the Steiner point create 120°
- Maximum number of Steiner points is N – 2

Let's take another look at the three point Steiner Tree Problem.



Solution:

The Knights of Win Racing Company should have a location around Atlanta Georgia to have the cheapest network.



One Algorithm to Solve three point problem:

Fermat-Torricelli point

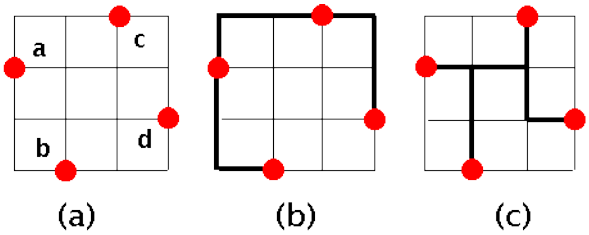
- Determine largest angle. Is it less than 120? Then we are going to use this algorithm
- Take the largest length and create an equilateral triangle
- Circumscribe the triangle and connect the third vertex to the third point of the original graph.
- The intersection of the circle and the line creates a Steiner point

More than three points:

- Break up into smaller parts using Melzak's algorithm
  - Determining which points to use first gets a little tricky
  - Computers can come into play but even they take time
- The Steiner Problem is a NP-hard problem
  - It's considered a complex problem
  - Optimal solution may not be found in polynomial time algorithm ("fast" algorithm)

Applications:

- Rectilinear Steiner Trees
  - Graph is on a computer chip grid to increase operating speeds
  - On a grid were not all properties of the Euclidean Steiner Tree problem apply
- Networks
  - Telephone companies
  - Pipelines
  - Roadways
  - Computer chips



When you get the time look up the soap bubble their cool!
https://www.youtube.com/watch?v=PI6rAOWu-Og